REVIEW OF ANALYTIC GEOMETRY

The points in a plane can be identified with ordered pairs of real numbers. We start by drawing two perpendicular coordinate lines that intersect at the origin O on each line. Usually one line is horizontal with positive direction to the right and is called the x-axis; the other line is vertical with positive direction upward and is called the y-axis.

Any point P in the plane can be located by a unique ordered pair of numbers as follows. Draw lines through P perpendicular to the x- and y-axes. These lines intersect the axes in points with coordinates a and b as shown in Figure 1. Then the point P is assigned the ordered pair (a, b). The first number a is called the ***x*-coordinate** of P; the second number b is called the ***y*-coordinate** of P. We say that P is the point with coordinates (a, b), and we denote the point by the symbol $P(a, b)$. Several points are labeled with their coordinates in Figure 2.

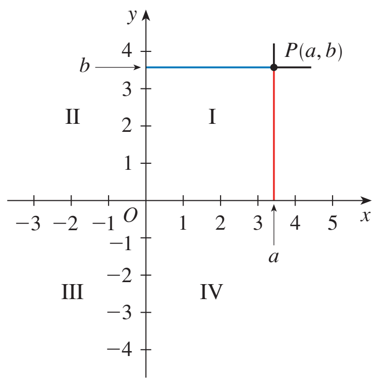

FIGURE 1

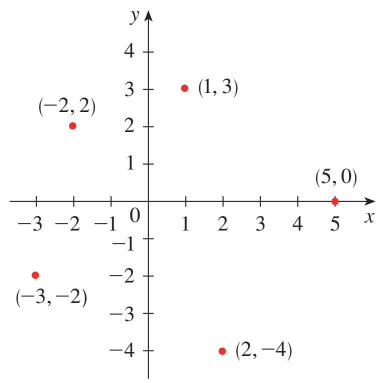

FIGURE 2

By reversing the preceding process we can start with an ordered pair (a, b) and arrive at the corresponding point P. Often we identify the point P with the ordered pair (a, b) and refer to "the point (a, b)." [Although the notation used for an open interval (a, b) is the same as the notation used for a point (a, b), you will be able to tell from the context which meaning is intended.]

This coordinate system is called the **rectangular coordinate system** or the **Cartesian coordinate system** in honor of the French mathematician René Descartes (1596–1650), even though another Frenchman, Pierre Fermat (1601–1665), invented the principles of analytic geometry at about the same time as Descartes. The plane supplied with this coordinate system is called the **coordinate plane** or the **Cartesian plane** and is denoted by $\mathbb{R}^2$.

The x- and y-axes are called the **coordinate axes** and divide the Cartesian plane into four quadrants, which are labeled I, II, III, and IV in Figure 1. Notice that the first quadrant consists of those points whose x- and y-coordinates are both positive.

EXAMPLE 1 Describe and sketch the regions given by the following sets.

(a) $\{(x, y) \mid x \geqslant 0\}$ (b) $\{(x, y) \mid y = 1\}$ (c) $\{(x, y) \mid |y| < 1\}$

SOLUTION

(a) The points whose x-coordinates are 0 or positive lie on the y-axis or to the right of it as indicated by the shaded region in Figure 3(a).

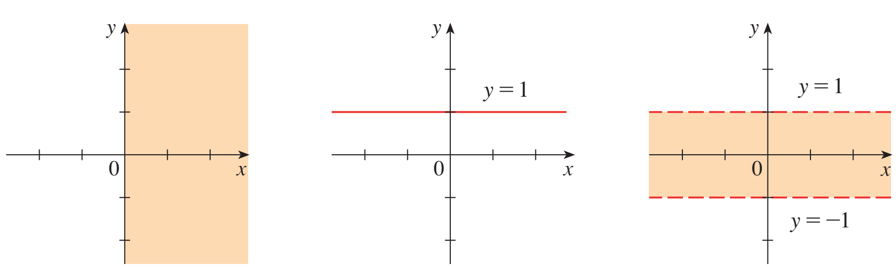

FIGURE 3 (a) $x \geqslant 0$ (b) $y = 1$ (c) $|y| < 1$

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(b) The set of all points with y-coordinate 1 is a horizontal line one unit above the x-axis [see Figure 3(b)].

(c) Recall from **Review of Algebra** that

$$|y| < 1 \qquad \text{if and only if} \qquad -1 < y < 1$$

The given region consists of those points in the plane whose y-coordinates lie between -1 and 1. Thus, the region consists of all points that lie between (but not on) the horizontal lines $y = 1$ and $y = -1$. [These lines are shown as dashed lines in Figure 3(c) to indicate that the points on these lines don't lie in the set.] ■

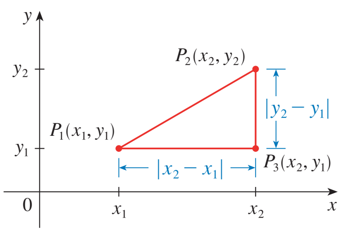

FIGURE 4

Recall from **Review of Algebra** that the distance between points a and b on a number line is $|a - b| = |b - a|$. Thus, the distance between points $P_1(x_1, y_1)$ and $P_3(x_2, y_1)$ on a horizontal line must be $|x_2 - x_1|$ and the distance between $P_2(x_2, y_2)$ and $P_3(x_2, y_1)$ on a vertical line must be $|y_2 - y_1|$. (See Figure 4.)

To find the distance $|P_1P_2|$ between any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, we note that triangle $P_1P_2P_3$ in Figure 4 is a right triangle, and so by the Pythagorean Theorem we have

$$\begin{aligned} |P_1P_2| &= \sqrt{|P_1P_3|^2 + |P_2P_3|^2} = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \\ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$

Distance Formula The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For instance, the distance between $(1, -2)$ and $(5, 3)$ is

$$\sqrt{(5 - 1)^2 + [3 - (-2)]^2} = \sqrt{4^2 + 5^2} = \sqrt{41}$$

CIRCLES

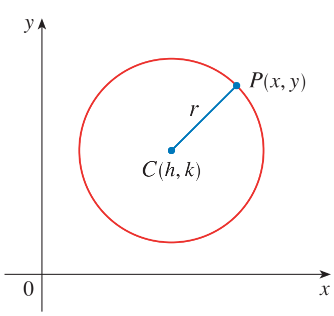

FIGURE 5

An **equation of a curve** is an equation satisfied by the coordinates of the points on the curve and by no other points. Let's use the distance formula to find the equation of a circle with radius r and center (h, k). By definition, the circle is the set of all points $P(x, y)$ whose distance from the center $C(h, k)$ is r. (See Figure 5.) Thus, P is on the circle if and only if $|PC| = r$. From the distance formula, we have

$$\sqrt{(x - h)^2 + (y - k)^2} = r$$

or equivalently, squaring both sides, we get

$$(x - h)^2 + (y - k)^2 = r^2$$

This is the desired equation.

Equation of a Circle An equation of the circle with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

In particular, if the center is the origin $(0, 0)$, the equation is

$$x^2 + y^2 = r^2$$

For instance, an equation of the circle with radius 3 and center $(2, -5)$ is

$$(x - 2)^2 + (y + 5)^2 = 9$$

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EXAMPLE 2 Sketch the graph of the equation $x^2 + y^2 + 2x - 6y + 7 = 0$ by first showing that it represents a circle and then finding its center and radius.

SOLUTION We first group the x-terms and y-terms as follows:

$$(x^2 + 2x) + (y^2 - 6y) = -7$$

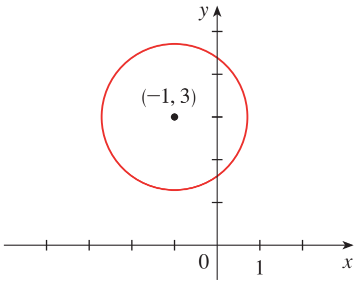

FIGURE 6
$x^2 + y^2 + 2x - 6y + 7 = 0$

Then we complete the square within each grouping, adding the appropriate constants (the squares of half the coefficients of x and y) to both sides of the equation:

$$(x^2 + 2x + 1) + (y^2 - 6y + 9) = -7 + 1 + 9$$

or
$$(x + 1)^2 + (y - 3)^2 = 3$$

Comparing this equation with the standard equation of a circle, we see that $h = -1$, $k = 3$, and $r = \sqrt{3}$, so the given equation represents a circle with center $(-1, 3)$ and radius $\sqrt{3}$. It is sketched in Figure 6. ■

LINES

To find the equation of a line L we use its *slope,* which is a measure of the steepness of the line.

> **Definition** The **slope** of a nonvertical line that passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is
>
> $$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$
>
> The slope of a vertical line is not defined.

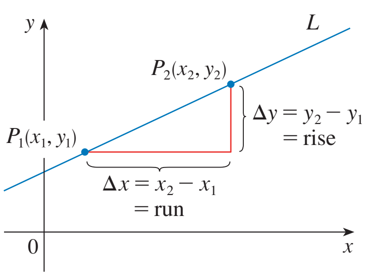

FIGURE 7

Thus the slope of a line is the ratio of the change in y, Δy, to the change in x, Δx. (See Figure 7.) The slope is therefore the rate of change of y with respect to x. The fact that the line is straight means that the rate of change is constant.

Figure 8 shows several lines labeled with their slopes. Notice that lines with positive slope slant upward to the right, whereas lines with negative slope slant downward to the right. Notice also that the steepest lines are the ones for which the absolute value of the slope is largest, and a horizontal line has slope 0.

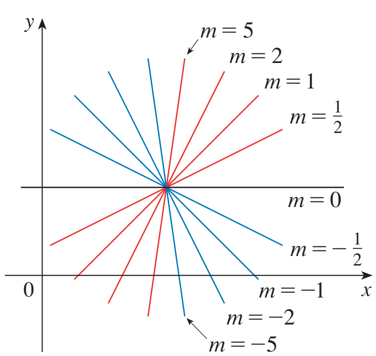

FIGURE 8

Now let's find an equation of the line that passes through a given point $P_1(x_1, y_1)$ and has slope m. A point $P(x, y)$ with $x \neq x_1$ lies on this line if and only if the slope of the line through P_1 and P is equal to m; that is,

$$\frac{y - y_1}{x - x_1} = m$$

This equation can be rewritten in the form

$$y - y_1 = m(x - x_1)$$

and we observe that this equation is also satisfied when $x = x_1$ and $y = y_1$. Therefore, it is an equation of the given line.

> **Point-Slope Form of the Equation of a Line** An equation of the line passing through the point $P_1(x_1, y_1)$ and having slope m is
>
> $$y - y_1 = m(x - x_1)$$

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EXAMPLE 3 Find an equation of the line through the points $(-1, 2)$ and $(3, -4)$.

SOLUTION The slope of the line is

$$m = \frac{-4 - 2}{3 - (-1)} = -\frac{3}{2}$$

Using the point-slope form with $x_1 = -1$ and $y_1 = 2$, we obtain

$$y - 2 = -\tfrac{3}{2}(x + 1)$$

which simplifies to $$3x + 2y = 1$$ ■

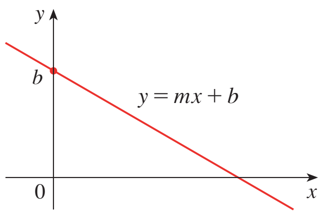

FIGURE 9

Suppose a nonvertical line has slope m and y-intercept b. (See Figure 9.) This means it intersects the y-axis at the point $(0, b)$, so the point-slope form of the equation of the line, with $x_1 = 0$ and $y_1 = b$, becomes

$$y - b = m(x - 0)$$

This simplifies as follows.

Slope-Intercept Form of the Equation of a Line An equation of the line with slope m and y-intercept b is

$$y = mx + b$$

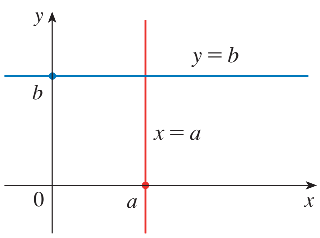

FIGURE 10

In particular, if a line is horizontal, its slope is $m = 0$, so its equation is $y = b$, where b is the y-intercept (see Figure 10). A vertical line does not have a slope, but we can write its equation as $x = a$, where a is the x-intercept, because the x-coordinate of every point on the line is a.

EXAMPLE 4 Graph the inequality $x + 2y > 5$.

SOLUTION We are asked to sketch the graph of the set $\{(x, y) \mid x + 2y > 5\}$ and we begin by solving the inequality for y:

$$x + 2y > 5$$
$$2y > -x + 5$$
$$y > -\tfrac{1}{2}x + \tfrac{5}{2}$$

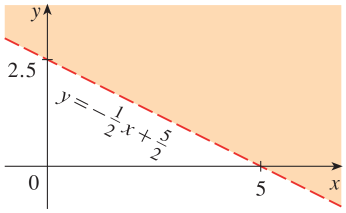

FIGURE 11

Compare this inequality with the equation $y = -\frac{1}{2}x + \frac{5}{2}$, which represents a line with slope $-\frac{1}{2}$ and y-intercept $\frac{5}{2}$. We see that the given graph consists of points whose y-coordinates are *larger* than those on the line $y = -\frac{1}{2}x + \frac{5}{2}$. Thus, the graph is the region that lies *above* the line, as illustrated in Figure 11. ■

PARALLEL AND PERPENDICULAR LINES

Slopes can be used to show that lines are parallel or perpendicular. The following facts are proved, for instance, in *Precalculus: Mathematics for Calculus, Fifth Edition* by Stewart, Redlin, and Watson (Thomson Brooks/Cole, Belmont, CA, 2006).

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Parallel and Perpendicular Lines

1. Two nonvertical lines are parallel if and only if they have the same slope.
2. Two lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$; that is, their slopes are negative reciprocals:

$$m_2 = -\frac{1}{m_1}$$

EXAMPLE 5 Find an equation of the line through the point $(5, 2)$ that is parallel to the line $4x + 6y + 5 = 0$.

SOLUTION The given line can be written in the form

$$y = -\tfrac{2}{3}x - \tfrac{5}{6}$$

which is in slope-intercept form with $m = -\frac{2}{3}$. Parallel lines have the same slope, so the required line has slope $-\frac{2}{3}$ and its equation in point-slope form is

$$y - 2 = -\tfrac{2}{3}(x - 5)$$

We can write this equation as $2x + 3y = 16$. ■

EXAMPLE 6 Show that the lines $2x + 3y = 1$ and $6x - 4y - 1 = 0$ are perpendicular.

SOLUTION The equations can be written as

$$y = -\tfrac{2}{3}x + \tfrac{1}{3} \qquad \text{and} \qquad y = \tfrac{3}{2}x - \tfrac{1}{4}$$

from which we see that the slopes are

$$m_1 = -\tfrac{2}{3} \qquad \text{and} \qquad m_2 = \tfrac{3}{2}$$

Since $m_1 m_2 = -1$, the lines are perpendicular. ■

EXERCISES

A Click here for answers.

1–2 ■ Find the distance between the points.

1. $(1, 1)$, $(4, 5)$ **2.** $(1, -3)$, $(5, 7)$

3–4 ■ Find the slope of the line through P and Q.

3. $P(-3, 3)$, $Q(-1, -6)$ **4.** $P(-1, -4)$, $Q(6, 0)$

5. Show that the points $(-2, 9)$, $(4, 6)$, $(1, 0)$, and $(-5, 3)$ are the vertices of a square.

6. (a) Show that the points $A(-1, 3)$, $B(3, 11)$, and $C(5, 15)$ are collinear (lie on the same line) by showing that $|AB| + |BC| = |AC|$.
(b) Use slopes to show that A, B, and C are collinear.

7–10 ■ Sketch the graph of the equation.

7. $x = 3$ **8.** $y = -2$

9. $xy = 0$ **10.** $|y| = 1$

11–24 ■ Find an equation of the line that satisfies the given conditions.

11. Through $(2, -3)$, slope 6

12. Through $(-3, -5)$, slope $-\frac{7}{2}$

13. Through $(2, 1)$ and $(1, 6)$

14. Through $(-1, -2)$ and $(4, 3)$

15. Slope 3, y-intercept -2

16. Slope $\frac{2}{5}$, y-intercept 4

17. x-intercept 1, y-intercept -3

18. x-intercept -8, y-intercept 6

19. Through $(4, 5)$, parallel to the x-axis

20. Through $(4, 5)$, parallel to the y-axis

21. Through $(1, -6)$, parallel to the line $x + 2y = 6$

22. y-intercept 6, parallel to the line $2x + 3y + 4 = 0$

23. Through $(-1, -2)$, perpendicular to the line $2x + 5y + 8 = 0$

24. Through $\left(\frac{1}{2}, -\frac{2}{3}\right)$, perpendicular to the line $4x - 8y = 1$

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25–28 ■ Find the slope and y-intercept of the line and draw its graph.

25. $x + 3y = 0$

26. $2x - 3y + 6 = 0$

27. $3x - 4y = 12$

28. $4x + 5y = 10$

29–36 ■ Sketch the region in the xy-plane.

29. $\{(x, y) \mid x < 0\}$

30. $\{(x, y) \mid x \geq 1 \text{ and } y < 3\}$

31. $\{(x, y) \mid |x| \leq 2\}$

32. $\{(x, y) \mid |x| < 3 \text{ and } |y| < 2\}$

33. $\{(x, y) \mid 0 \leq y \leq 4 \text{ and } x \leq 2\}$

34. $\{(x, y) \mid y > 2x - 1\}$

35. $\{(x, y) \mid 1 + x \leq y \leq 1 - 2x\}$

36. $\{(x, y) \mid -x \leq y < \frac{1}{2}(x + 3)\}$

37–38 ■ Find an equation of a circle that satisfies the given conditions.

37. Center $(3, -1)$, radius 5

38. Center $(-1, 5)$, passes through $(-4, -6)$

39–40 ■ Show that the equation represents a circle and find the center and radius.

39. $x^2 + y^2 - 4x + 10y + 13 = 0$

40. $x^2 + y^2 + 6y + 2 = 0$

41. Show that the lines

$$2x - y = 4 \qquad \text{and} \qquad 6x - 2y = 10$$

are not parallel and find their point of intersection.

42. Show that the lines

$$3x - 5y + 19 = 0 \qquad \text{and} \qquad 10x + 6y - 50 = 0$$

are perpendicular and find their point of intersection.

43. Show that the midpoint of the line segment from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

44. Find the midpoint of the line segment joining the points $(1, 3)$ and $(7, 15)$.

45. Find an equation of the perpendicular bisector of the line segment joining the points $A(1, 4)$ and $B(7, -2)$.

46. (a) Show that if the x- and y-intercepts of a line are nonzero numbers a and b, then the equation of the line can be put in the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

This equation is called the **two-intercept form** of an equation of a line.

(b) Use part (a) to find an equation of the line whose x-intercept is 6 and whose y-intercept is -8.

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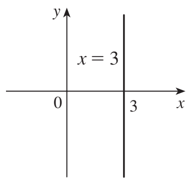
ANSWERS

1. 5 **2.** $2\sqrt{29}$ **3.** $-\frac{9}{2}$ **4.** $\frac{4}{7}$

7.



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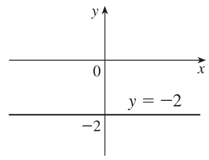

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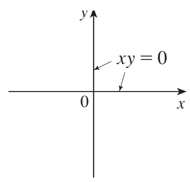

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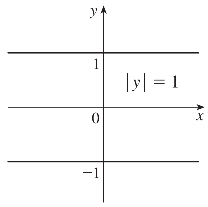


11. $y = 6x - 15$ **12.** $y = -\frac{7}{2}x - \frac{31}{2}$ **13.** $5x + y = 11$

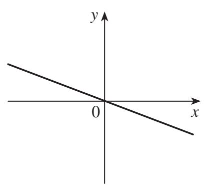
14. $y = x - 1$ **15.** $y = 3x - 2$ **16.** $y = \frac{2}{5}x + 4$

17. $y = 3x - 3$ **18.** $y = \frac{3}{4}x + 6$ **19.** $y = 5$

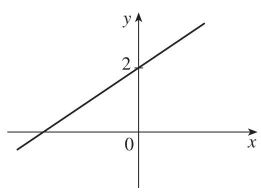
20. $x = 4$ **21.** $x + 2y + 11 = 0$ **22.** $y = -\frac{2}{3}x + 6$

23. $5x - 2y + 1 = 0$ **24.** $y = -2x + \frac{1}{3}$

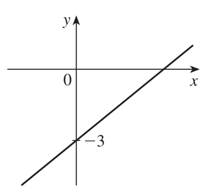
25. $m = -\frac{1}{3}, \quad b = 0$



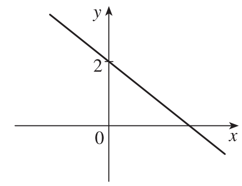
26. $m = \frac{2}{3}, \quad b = 2$



27. $m = \frac{3}{4}, \quad b = -3$



28. $m = -\frac{4}{5}, \quad b = 2$



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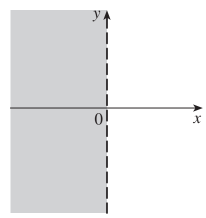

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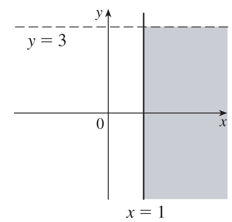

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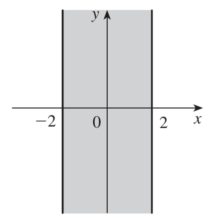

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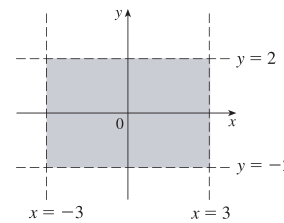

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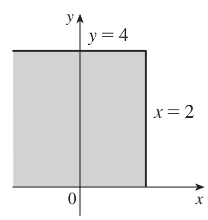

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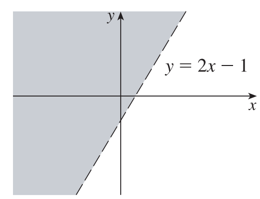

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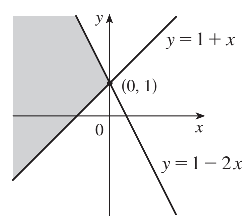

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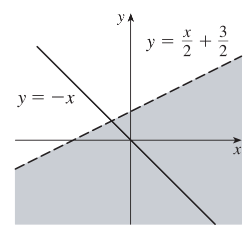

37. $(x - 3)^2 + (y + 1)^2 = 25$ **38.** $(x + 1)^2 + (y - 5)^2 = 130$

39. $(2, -5)$, 4 **40.** $(0, -3)$, $\sqrt{7}$ **41.** $(1, -2)$ **42.** $(2, 5)$

44. $(4, 9)$ **45.** $y = x - 3$ **46.** (b) $y = \frac{4}{3}x - 8$

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SOLUTIONS

1. Use the distance formula with $P_1(x_1, y_1) = (1, 1)$ and $P_2(x_2, y_2) = (4, 5)$ to get

$$|P_1P_2| = \sqrt{(4-1)^2 + (5-1)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

2. The distance from $(1, -3)$ to $(5, 7)$ is $\sqrt{(5-1)^2 + [7-(-3)]^2} = \sqrt{4^2 + 10^2} = \sqrt{116} = 2\sqrt{29}$.

3. With $P(-3, 3)$ and $Q(-1, -6)$, the slope m of the line through P and Q is $m = \dfrac{-6-3}{-1-(-3)} = -\dfrac{9}{2}$.

4. $m = \dfrac{0-(-4)}{6-(-1)} = \dfrac{4}{7}$

5. Using $A(-2, 9)$, $B(4, 6)$, $C(1, 0)$, and $D(-5, 3)$, we have

$|AB| = \sqrt{[4-(-2)]^2 + (6-9)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$,

$|BC| = \sqrt{(1-4)^2 + (0-6)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$,

$|CD| = \sqrt{(-5-1)^2 + (3-0)^2} = \sqrt{(-6)^2 + 3^2} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$, and

$|DA| = \sqrt{[-2-(-5)]^2 + (9-3)^2} = \sqrt{3^2 + 6^2} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$. So all sides are of equal length and we have a rhombus. Moreover, $m_{AB} = \dfrac{6-9}{4-(-2)} = -\dfrac{1}{2}$, $m_{BC} = \dfrac{0-6}{1-4} = 2$, $m_{CD} = \dfrac{3-0}{-5-1} = -\dfrac{1}{2}$, and $m_{DA} = \dfrac{9-3}{-2-(-5)} = 2$, so the sides are perpendicular. Thus, A, B, C, and D are vertices of a square.

6. (a) Using $A(-1, 3)$, $B(3, 11)$, and $C(5, 15)$, we have

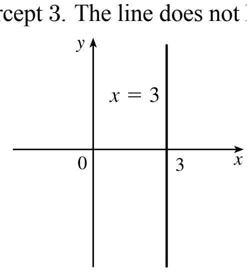
$|AB| = \sqrt{[3-(-1)]^2 + (11-3)^2} = \sqrt{4^2 + 8^2} = \sqrt{80} = 4\sqrt{5}$,

$|BC| = \sqrt{(5-3)^2 + (15-11)^2} = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$, and

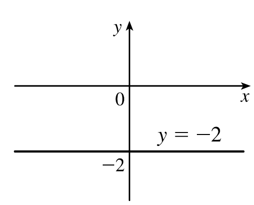
$|AC| = \sqrt{[5-(-1)]^2 + (15-3)^2} = \sqrt{6^2 + 12^2} = \sqrt{180} = 6\sqrt{5}$. Thus, $|AC| = |AB| + |BC|$.

(b) $m_{AB} = \dfrac{11-3}{3-(-1)} = \dfrac{8}{4} = 2$ and $m_{AC} = \dfrac{15-3}{5-(-1)} = \dfrac{12}{6} = 2$. Since the segments AB and AC have the same slope, A, B and C must be collinear.

7. The graph of the equation $x = 3$ is a vertical line with x-intercept 3. The line does not have a slope.



8. The graph of the equation $y = -2$ is a horizontal line with y-intercept -2. The line has slope 0.



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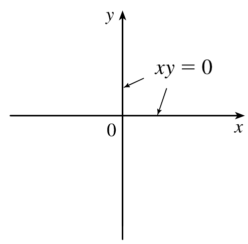
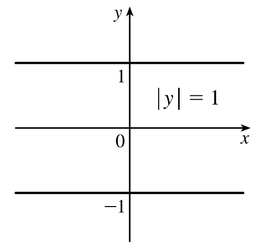
9. $xy = 0 \quad \Leftrightarrow \quad x = 0$ or $y = 0$. The graph consists of the coordinate axes.



10. $|y| = 1 \quad \Leftrightarrow \quad y = 1$ or $y = -1$



11. By the point-slope form of the equation of a line, an equation of the line through $(2, -3)$ with slope 6 is $y - (-3) = 6(x - 2)$ or $y = 6x - 15$.

12. $y - (-5) = -\frac{7}{2}\,[x - (-3)]$ or $y = -\frac{7}{2}x - \frac{31}{2}$

13. The slope of the line through $(2, 1)$ and $(1, 6)$ is $m = \dfrac{6 - 1}{1 - 2} = -5$, so an equation of the line is $y - 1 = -5(x - 2)$ or $y = -5x + 11$.

14. For $(-1, -2)$ and $(4, 3)$, $m = \dfrac{3 - (-2)}{4 - (-1)} = 1$. An equation of the line is $y - 3 = 1(x - 4)$ or $y = x - 1$.

15. By the slope-intercept form of the equation of a line, an equation of the line is $y = 3x - 2$.

16. By the slope-intercept form of the equation of a line, an equation of the line is $y = \frac{2}{5}x + 4$.

17. Since the line passes through $(1, 0)$ and $(0, -3)$, its slope is $m = \dfrac{-3 - 0}{0 - 1} = 3$, so an equation is $y = 3x - 3$.

Another method: From Exercise 61, $\dfrac{x}{1} + \dfrac{y}{-3} = 1 \quad \Rightarrow \quad -3x + y = -3 \quad \Rightarrow \quad y = 3x - 3$.

18. For $(-8, 0)$ and $(0, 6)$, $m = \dfrac{6 - 0}{0 - (-8)} = \dfrac{3}{4}$. So an equation is $y = \frac{3}{4}x + 6$.

Another method: From Exercise 61, $\dfrac{x}{-8} + \dfrac{y}{6} = 1 \quad \Rightarrow \quad -3x + 4y = 24 \quad \Rightarrow \quad y = \frac{3}{4}x + 6$.

19. The line is parallel to the x-axis, so it is horizontal and must have the form $y = k$. Since it goes through the point $(x, y) = (4, 5)$, the equation is $y = 5$.

20. The line is parallel to the y-axis, so it is vertical and must have the form $x = k$. Since it goes through the point $(x, y) = (4, 5)$, the equation is $x = 4$.

21. Putting the line $x + 2y = 6$ into its slope-intercept form gives us $y = -\frac{1}{2}x + 3$, so we see that this line has slope $-\frac{1}{2}$. Thus, we want the line of slope $-\frac{1}{2}$ that passes through the point $(1, -6)$: $y - (-6) = -\frac{1}{2}(x - 1) \quad \Leftrightarrow$ $y = -\frac{1}{2}x - \frac{11}{2}$.

22. $2x + 3y + 4 = 0 \quad \Leftrightarrow \quad y = -\frac{2}{3}x - \frac{4}{3}$, so $m = -\frac{2}{3}$ and the required line is $y = -\frac{2}{3}x + 6$.

23. $2x + 5y + 8 = 0 \quad \Leftrightarrow \quad y = -\frac{2}{5}x - \frac{8}{5}$. Since this line has slope $-\frac{2}{5}$, a line perpendicular to it would have slope $\frac{5}{2}$, so the required line is $y - (-2) = \frac{5}{2}\,[x - (-1)] \quad \Leftrightarrow \quad y = \frac{5}{2}x + \frac{1}{2}$.

24. $4x - 8y = 1 \quad \Leftrightarrow \quad y = \frac{1}{2}x - \frac{1}{8}$. Since this line has slope $\frac{1}{2}$, a line perpendicular to it would have slope -2, so the required line is $y - \left(-\frac{2}{3}\right) = -2\left(x - \frac{1}{2}\right) \quad \Leftrightarrow \quad y = -2x + \frac{1}{3}$.

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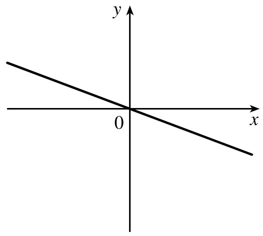
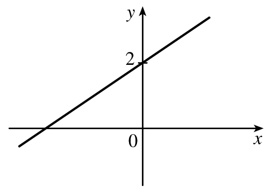
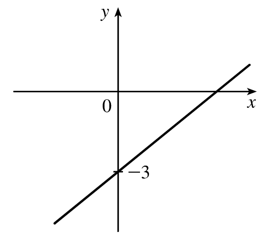
25. $x + 3y = 0 \quad \Leftrightarrow \quad y = -\frac{1}{3}x$, so the slope is $-\frac{1}{3}$ and the y-intercept is 0.



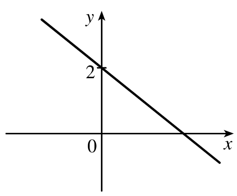
26. $2x - 3y + 6 = 0 \quad \Leftrightarrow$ $y = \frac{2}{3}x + 2$, so the slope is $\frac{2}{3}$ and the y-intercept is 2.



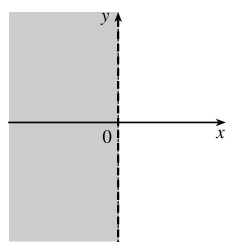
27. $3x - 4y = 12 \quad \Leftrightarrow$ $y = \frac{3}{4}x - 3$, so the slope is $\frac{3}{4}$ and the y-intercept is -3.



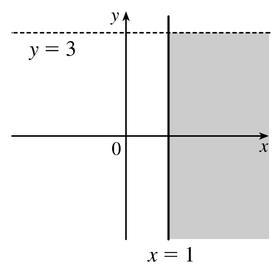
28. $4x + 5y = 10 \quad \Leftrightarrow$ $y = -\frac{4}{5}x + 2$, so the slope is $-\frac{4}{5}$ and the y-intercept is 2.



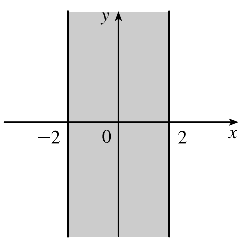
29. $\{(x, y) \mid x < 0\}$



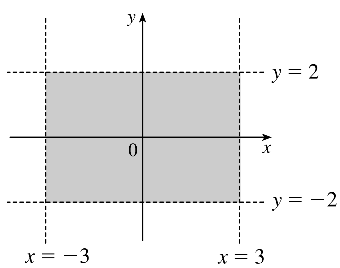
30. $\{(x, y) \mid x \geq 1 \text{ and } y < 3\}$



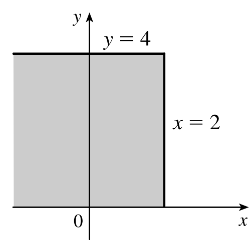
31. $\left\{(x, y) \,\middle|\, |x| \leq 2\right\} =$ $\{(x, y) \mid -2 \leq x \leq 2\}$



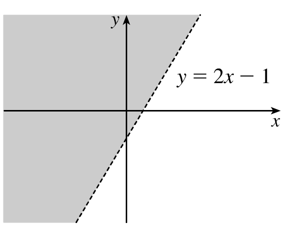
32. $\left\{(x, y) \,\middle|\, |x| < 3 \text{ and } |y| < 2\right\}$



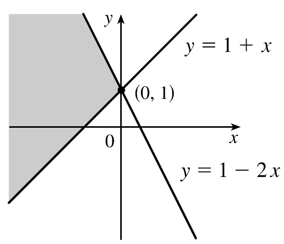
33. $\{(x, y) \mid 0 \leq y \leq 4, x \leq 2\}$



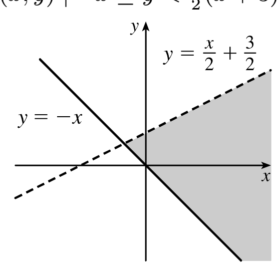
34. $\{(x, y) \mid y > 2x - 1\}$



35. $\{(x, y) \mid 1 + x \leq y \leq 1 - 2x\}$



36. $\left\{(x, y) \mid -x \leq y < \frac{1}{2}(x + 3)\right\}$



37. An equation of the circle with center $(3, -1)$ and radius 5 is $(x - 3)^2 + (y + 1)^2 = 5^2 = 25$.

38. The equation has the form $(x + 1)^2 + (y - 5)^2 = r^2$. Since $(-4, -6)$ lies on the circle, we have $r^2 = (-4 + 1)^2 + (-6 - 5)^2 = 130$. So an equation is $(x + 1)^2 + (y - 5)^2 = 130$.

39. $x^2 + y^2 - 4x + 10y + 13 = 0 \quad \Leftrightarrow \quad x^2 - 4x + y^2 + 10y = -13 \quad \Leftrightarrow$ $\left(x^2 - 4x + 4\right) + \left(y^2 + 10y + 25\right) = -13 + 4 + 25 = 16 \quad \Leftrightarrow \quad (x - 2)^2 + (y + 5)^2 = 4^2$. Thus, we have a circle with center $(2, -5)$ and radius 4.

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40. $x^2 + y^2 + 6y + 2 = 0 \quad \Leftrightarrow \quad x^2 + \left(y^2 + 6y + 9\right) = -2 + 9 \quad \Leftrightarrow \quad x^2 + (y+3)^2 = 7$. Thus, we have a circle with center $(0, -3)$ and radius $\sqrt{7}$.

41. $2x - y = 4 \quad \Leftrightarrow \quad y = 2x - 4 \quad \Rightarrow \quad m_1 = 2$ and $6x - 2y = 10 \quad \Leftrightarrow \quad 2y = 6x - 10 \quad \Leftrightarrow \quad y = 3x - 5 \quad \Rightarrow$ $m_2 = 3$. Since $m_1 \neq m_2$, the two lines are not parallel. To find the point of intersection: $2x - 4 = 3x - 5 \quad \Leftrightarrow$ $x = 1 \quad \Rightarrow \quad y = -2$. Thus, the point of intersection is $(1, -2)$.

42. $3x - 5y + 19 = 0 \quad \Leftrightarrow \quad 5y = 3x + 19 \quad \Leftrightarrow \quad y = \frac{3}{5}x + \frac{19}{5} \quad \Rightarrow \quad m_1 = \frac{3}{5}$ and $10x + 6y - 50 = 0 \quad \Leftrightarrow$ $6y = -10x + 50 \quad \Leftrightarrow \quad y = -\frac{5}{3}x + \frac{25}{3} \quad \Rightarrow \quad m_2 = -\frac{5}{3}$. Since $m_1 m_2 = \frac{3}{5}\left(-\frac{5}{3}\right) = -1$, the two lines are perpendicular. To find the point of intersection: $\frac{3}{5}x + \frac{19}{5} = -\frac{5}{3}x + \frac{25}{3} \quad \Leftrightarrow \quad 9x + 57 = -25x + 125 \quad \Leftrightarrow$ $34x = 68 \quad \Leftrightarrow \quad x = 2 \quad \Rightarrow \quad y = \frac{3}{5} \cdot 2 + \frac{19}{5} = \frac{25}{5} = 5$. Thus, the point of intersection is $(2, 5)$.

43. Let M be the point $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$. Then

$$|MP_1|^2 = \left(x_1 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_1 - \frac{y_1 + y_2}{2}\right)^2 = \left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2 \text{ and}$$

$$|MP_2|^2 = \left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 + y_2}{2}\right)^2 = \left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2.$$

Hence, $|MP_1| = |MP_2|$; that is, M is equidistant from P_1 and P_2.

44. Using the midpoint formula from Exercise 43 with $(1, 3)$ and $(7, 15)$, we get $\left(\frac{1+7}{2}, \frac{3+15}{2}\right) = (4, 9)$.

45. With $A(1, 4)$ and $B(7, -2)$, the slope of segment AB is $\frac{-2-4}{7-1} = -1$, so its perpendicular bisector has slope 1. The midpoint of AB is $\left(\frac{1+7}{2}, \frac{4+(-2)}{2}\right) = (4, 1)$, so an equation of the perpendicular bisector is $y - 1 = 1(x - 4)$ or $y = x - 3$.

46. (a) Since the x-intercept is a, the point $(a, 0)$ is on the line, and similarly since the y-intercept is b, $(0, b)$ is on the line. Hence, the slope of the line is $m = \dfrac{b - 0}{0 - a} = -\dfrac{b}{a}$. Substituting into $y = mx + b$ gives $y = -\dfrac{b}{a}x + b \quad \Leftrightarrow$ $\dfrac{b}{a}x + y = b \quad \Leftrightarrow \quad \dfrac{x}{a} + \dfrac{y}{b} = 1$.

(b) Letting $a = 6$ and $b = -8$ gives $\dfrac{x}{6} + \dfrac{y}{-8} = 1 \quad \Leftrightarrow \quad -8x + 6y = -48$ [multiply by -48] $\quad \Leftrightarrow$ $6y = 8x - 48 \quad \Leftrightarrow \quad 3y = 4x - 24 \quad \Leftrightarrow \quad y = \frac{4}{3}x - 8$.

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